CBSE 2026 EXAMINATIONS
XII MATHEMATICS (041)
Series SR3PQ ♦ Q.P. Code 65/3/1; 65/3/2; 65/3/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **03 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/3/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. The domain of $f(x)=\cos^{-1}(2x-5)$ is

(a) $[-1, 1]$ (b) $[4, 6]$ (c) $[-7, -3]$ (d) $[2, 3]$

Sol. (d) $f(x)=\cos^{-1}(2x-5)$ is defined when $-1\le(2x-5)\le1$

$\Rightarrow -1+5\le(2x-5)+5\le1+5$

$\Rightarrow 4\le(2x)\le6$

$\Rightarrow 2\le x\le3$

$\therefore \text{Dom.}(f)=x\in[2, 3]$.

02. If $A^2=4A+3I$ and $A^{-1}=xA+yI$, then the value of $(x+y)$ is

(a) -1 (b) 1 (c) $\frac{5}{3}$ (d) 7

Sol. (a) $\because A^2=4A+3I \Rightarrow A^{-1}AA=4A^{-1}A+3A^{-1}I$

$\Rightarrow IA=4I+3A^{-1}$

$\Rightarrow A^{-1}=\frac{1}{3}A-\frac{4}{3}I$

On comparing with $A^{-1}=xA+yI$, we get $x=\frac{1}{3}$, $y=-\frac{4}{3}$

Hence, $(x+y)=\frac{1}{3}+\frac{-4}{3}=\frac{-3}{3}=-1$.

03. If A and B are skew-symmetric matrices of same order, then $AB'+BA'$ is a/an

(a) symmetric matrix (b) skew-symmetric matrix
(c) null matrix (d) identity matrix

Sol. (a) Let $P=AB'+BA'$

$\Rightarrow P'=(AB'+BA')'=(AB')'+(BA')'=BA'+AB'=AB'+BA'=P$

So, $P = AB' + BA'$ is a symmetric matrix.

04. If $\begin{bmatrix}4\\1\\3\end{bmatrix} A = \begin{bmatrix}-4 & 8 & 4\\-1 & 2 & 1\\-3 & 6 & 3\end{bmatrix}$, then order of A must be

(a) 3×1 (b) 1×3 (c) 1×1 (d) 3×3

Sol. (b) Since $\begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1} A_{m\times n} = \begin{bmatrix}-4 & 8 & 4\\-1 & 2 & 1\\-3 & 6 & 3\end{bmatrix}_{3\times3}$ so, clearly $m=1,\ n=3$.

Hence, order of A is $m\times n$ i.e., 1×3.

05. If a square matrix A is such that $A^2 = A$ and $(I-A)^3 = xA + I$, then value of x must be

(a) 7 (b) 5 (c) -7 (d) -1

Sol. (d) Since the identity matrix I commutes with a square matrix A (i.e., $AI = IA = A$), we can use the identity $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Now $(I-A)^3 = xA + I$

$\Rightarrow I^3 - 3I^2A + 3IA^2 - A^3 = xA + I$

$\Rightarrow I - 3IA + 3IA - A = xA + I$ [$\because A^2 = A \ \therefore A^3 = A^2A = AA = A^2 = A$

$\Rightarrow -A + I = xA + I$

On comparing both the sides, we get $x = -1$.

06. If $B(\text{adj.}B) = \begin{bmatrix}\frac{1}{3} & 0 & 0\\0 & \frac{1}{3} & 0\\0 & 0 & \frac{1}{3}\end{bmatrix}$, then the value of $\det(B^{-1}) =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) 3 (d) 9

Sol. (c) Since $B(\text{adj.}B) = |B|I = \frac{1}{3}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \Rightarrow |B| = \frac{1}{3}$

Thus, $\det(B^{-1}) = [\det(B)]^{-1} = \left(\frac{1}{3}\right)^{-1} = 3$.

07. The value of k for which the function $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is

(a) $\frac{1}{4}$ (b) 2 (c) $\frac{1}{2}$ (d) 0

Sol. (d) As f is continuous so, $f(0) = \lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^+} f(x)$.

$\therefore k(0+1) = \lim\limits_{x\to0^-} x^2 \sin\frac{1}{x}$

$\Rightarrow k = 0^2 \times$ (an oscillating value lying between -1 and 1)

$\therefore k = 0$

Note that, range of $\sin\frac{1}{x}$ is $[-1, 1]$ for all $x \in \mathbb{R}$.

08. If $\sin^{-1} x = y$, then $\frac{dy}{dx}$ is

(a) $\cos^{-1} x$ (b) $\cos y$ (c) $\frac{1}{1-x^2}$ (d) $\sec y$

Sol. (d) For $\sin^{-1} x = y \quad \Rightarrow x = \sin y$

On differentiating w.r.t. y, $\frac{dx}{dy} = \cos y$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \sec y$.

09. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is

(a) 400π cm^3/cm (b) 100π cm^3/cm (c) 50π cm^3/cm (d) 25π cm^3/cm

Sol. (c) Volume of sphere with radius r is $V = \frac{4\pi}{3}r^3 = \frac{4\pi}{3}\left(\frac{s}{2}\right)^3$, where $s = 2r = \text{diameter}$

$\therefore \frac{dV}{ds} = \frac{4\pi}{3} \times \frac{1}{8} \times 3s^2$

$\Rightarrow \left(\frac{dV}{ds}\right)_{\text{at } s=10 \text{ cm}} = \frac{\pi}{2} \times (10)^2 = 50\ \pi \text{ cm}^3/\text{cm}$ $\quad [\because s = 2r = 2 \times 5 = 10 \text{ cm}$

10. $\int \frac{dx}{2^x + 2^{-x}}$ is equal to

(a) $\tan^{-1}(2^x) + C$ (b) $\tan^{-1}(2^{-x}) + C$ (c) $\frac{\tan^{-1}(2^x)}{\log 2} + C$ (d) $(\log 2)\tan^{-1}(2^x) + C$

Sol. (c) $\int \frac{dx}{2^x + 2^{-x}} = \int \frac{2^x dx}{(2^x)^2 + 1} = \frac{1}{\log 2}\int \frac{dt}{t^2+1}$ $\quad \left[\text{Put } 2^x = t \Rightarrow 2^x dx = \frac{dt}{\log 2}\right.$

$= \frac{1}{\log 2}\tan^{-1} t + C$

$= \frac{1}{\log 2}\tan^{-1} 2^x + C$.

11. $\int_{-1}^{1} (1 - |x|)\,dx$ is equal to

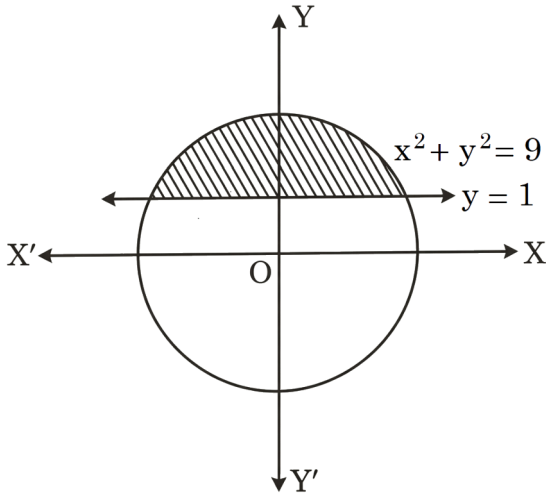
(a) $2\int_0^1 (1+x)\,dx$ (b) $2\int_{-1}^0 (1+x)\,dx$ (c) 0 (d) $2\int_{-1}^0 (1-x)\,dx$

Sol. (b) Note that, $f(x) = 1 - |x| = 1 - |-x| = f(-x)$ $\therefore f$ is an even function.

Therefore, $\int_{-1}^{1} (1-|x|)\,dx = 2\int_0^1 (1-|x|)\,dx = 2\int_{-1}^0 (1-|x|)\,dx = 2\int_{-1}^0 (1+x)\,dx$ $\quad [\because |x| = -x, \text{ if } x < 0$

Note that, $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$, also by the property of even functions, the area from $[-1, 0]$ is same as the area from $[0, 1]$ i.e., $\int_{-1}^0 (1-|x|)\,dx = \int_0^1 (1-|x|)\,dx$ or $\int_{-1}^0 (1+x)dx = \int_0^1 (1-x)dx$.

12. The area of the shaded region of the circle given below is equal to



(a) $\int_1^3 \sqrt{9-y^2}\,dy$ (b) $2\int_1^3 \sqrt{9-y^2}\,dy$ (c) $\int_0^3 \sqrt{9-x^2}\,dx$ (d) $2\int_0^3 \sqrt{9-x^2}\,dx$

Sol. (b) For $x^2 + y^2 = 9$, $x = \pm\sqrt{9-y^2}$. Also the circle is symmetrical about both the axes. Further its radius is of 3 units and centre at (0, 0).

Therefore, the required shaded area $= 2\int_1^3 x\,dy = 2\int_1^3 \sqrt{9-y^2}\,dy$.

13. $\frac{dy}{dx} = F(x,y)$ will be a homogeneous differential equation for which of the following functions?

(i) $F(x,y) = 3x + 2y$ (ii) $F(x,y) = \sin\frac{y}{x} + \log y - \log x$

(iii) $F(x,y) = e^{y/x} + 1$ (iv) $F(x,y) = \sqrt{x^2+y^2} - y$

(a) (i) and (ii) (b) (i), (ii) and (iii) (c) (ii), (iii) and (iv) (d) (ii) and (iii)

Sol. (d) Note that, a differential equation of the form $\frac{dy}{dx} = F(x,y)$ is said to be homogeneous if $F(x,y)$ is a homogeneous function of degree zero (0).

- (i) $F(\lambda x, \lambda y) = 3\lambda x + 2\lambda y = \lambda(3x+2y) = \lambda F(x,y)$
- (ii) $F(\lambda x, \lambda y) = \sin\frac{\lambda y}{\lambda x} + \log\frac{\lambda y}{\lambda x} = \sin\frac{y}{x} + \log\frac{y}{x} = \lambda^0 F(x,y)$
- (iii) $F(\lambda x, \lambda y) = e^{\lambda y/\lambda x} + 1 = e^{y/x} + 1 = \lambda^0 F(x,y)$
- (ii) $F(\lambda x, \lambda y) = \sqrt{\lambda^2x^2 + \lambda^2y^2} - \lambda y = \lambda\sqrt{x^2+y^2} - \lambda y = \lambda\left[\sqrt{x^2+y^2} - y\right] = \lambda F(x,y)$

Clearly, (i) and (iv) are homogeneous functions of degree 1, whereas (ii) and (iii) are the homogeneous functions of degree 0.
So, the differential equations in (ii) and (iii) are homogeneous.

14. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

(a) $\vec{a}.\vec{b} \le |\vec{a}||\vec{b}|$ (b) $|\vec{a}+\vec{b}| \ge |\vec{a}| + |\vec{b}|$ (c) $|\vec{a}-\vec{b}| = |\vec{a}| - |\vec{b}|$ (d) $|\vec{a}\times\vec{b}| \ge |\vec{a}||\vec{b}|$

Sol. (a) $\vec{a}.\vec{b} \le |\vec{a}||\vec{b}|$ is always true.

Recall that, $-1 \le \cos\theta \le 1 \Rightarrow -|\vec{a}||\vec{b}| \le |\vec{a}||\vec{b}|\cos\theta \le |\vec{a}||\vec{b}| \Rightarrow -|\vec{a}||\vec{b}| \le \vec{a}.\vec{b} \le |\vec{a}||\vec{b}|$.

- $|\vec{a}+\vec{b}| \ge |\vec{a}| + |\vec{b}|$ is false. According to the Triangle Inequality, $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$ is true.

- $|\vec{a}-\vec{b}|=|\vec{a}|-|\vec{b}|$ is false. The magnitude of the difference depends on the angle between the vectors. It is only equal to the difference of magnitudes if the vectors are collinear and point in the same direction and $|\vec{a}|\geq|\vec{b}|$. In general, $|\vec{a}-\vec{b}|\geq\big||\vec{a}|-|\vec{b}|\big|$.
- $|\vec{a}\times\vec{b}|\geq|\vec{a}||\vec{b}|$ is false.

Note that, $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$. Since the maximum value of $\sin\theta$ is 1.

So, $\sin\theta\leq 1 \Rightarrow |\vec{a}||\vec{b}|\sin\theta\leq|\vec{a}||\vec{b}| \Rightarrow |\vec{a}\times\vec{b}|\leq|\vec{a}||\vec{b}|$.

15. If $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=198$ and $|\vec{a}|=10|\vec{b}|$, then

(a) $|\vec{a}|=\sqrt{2}$ (b) $|\vec{b}|=\sqrt{2}$ (c) $|\vec{b}|=10\sqrt{2}$ (d) $|\vec{a}|=\dfrac{10}{\sqrt{2}}$

Sol. (b) $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=198 \Rightarrow |\vec{a}|^2-|\vec{b}|^2=198$

$\Rightarrow \left[10|\vec{b}|\right]^2-|\vec{b}|^2=198$

$\Rightarrow 100|\vec{b}|^2-|\vec{b}|^2=198$

$\Rightarrow 99|\vec{b}|^2=198$

$\Rightarrow |\vec{b}|^2=2 \quad \therefore |\vec{b}|=\sqrt{2}$.

16. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then

(a) $\cos\theta=l_1l_2+m_1m_2+n_1n_2$ (b) $\sin\theta=l_1l_2+m_1m_2+n_1n_2$

(c) $\tan\theta=\dfrac{l_1}{l_2}+\dfrac{m_1}{m_2}+\dfrac{n_1}{n_2}$ (d) $\cos\theta=|l_1l_2+m_1m_2+n_1n_2|$

Sol. (d) $\cos\theta=|l_1l_2+m_1m_2+n_1n_2|$, as θ is the acute angle between the lines L_1 and L_2.

17. Direction ratios of lines l_1 and l_2 are $<12,-3,9>$ and $<4,q,-p>$ respectively. The values of p and q for which l_1 and l_2 are parallel are respectively

(a) –1, 3 (b) 3, 1 (c) –3, –1 (d) –1, –3

Sol. (c) For parallel lines, their d.r.'s are proportional.

$\therefore \dfrac{12}{4}=\dfrac{-3}{q}=\dfrac{9}{-p}$

Consider $\dfrac{12}{4}=\dfrac{-3}{q}$ and $\dfrac{12}{4}=\dfrac{9}{-p}$

$\Rightarrow q=-1$ and $p=-3$.

18. If E and F are two independent events such that $P(E)=\dfrac{3}{10}$, $P(E\cup F)=\dfrac{1}{2}$, then $P(E|F)-P(F|E)$ is equal to

(a) $\dfrac{2}{7}$ (b) $\dfrac{3}{35}$ (c) $\dfrac{1}{70}$ (d) $\dfrac{1}{7}$

Sol. (c) As E and F are independent events so, $P(E\cap F)=P(E)P(F)$.

Also $P(E \cup F) = P(E) + P(F) - P(E \cap F) \quad \Rightarrow \frac{1}{2} = \frac{3}{10} + P(F) - P(E)\,P(F)$

$\Rightarrow \frac{2}{10} = P(F)\left[1 - P(E)\right] \quad \Rightarrow \frac{2}{10} = P(F)\left[1 - \frac{3}{10}\right] \quad \Rightarrow \frac{2}{10} = P(F) \times \frac{7}{10} \quad \Rightarrow P(F) = \frac{2}{7}$

Now $P(E \mid F) - P(F \mid E) = \frac{P(E \cap F)}{P(F)} - \frac{P(F \cap E)}{P(E)} = \frac{P(E)\,P(F)}{P(F)} - \frac{P(F)\,P(E)}{P(E)} = P(E) - P(F)$

$= \frac{3}{10} - \frac{2}{7} = \frac{21 - 20}{70} = \frac{1}{70}$.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R) : $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

Sol. (d) In the Assertion, it is mentioned that $e^x + e^{-y} = -2$ is a particular solution of $\frac{dy}{dx} = e^{x+y}$.

However, for any real values of x and y, $e^x > 0$ and $e^{-y} > 0$ i.e., both are always positive.
Therefore, their sum **must be greater than 0**. But $e^x + e^{-y} = -2$ is a negative number.
Hence, Assertion is false.

In the Reason, $\frac{dy}{dx} = e^{x+y} \quad \Rightarrow \frac{dy}{dx} = e^x \times e^y$

$\Rightarrow \int e^{-y} dy = \int e^x dx$

$\Rightarrow \frac{e^{-y}}{-1} = e^x + C$

$\therefore e^x + e^{-y} = C$ is the general solution of $\frac{dy}{dx} = e^{x+y}$.

Hence, Reason is true.

20. **Assertion (A) :** The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors.
Reason (R) : $\vec{a} \cdot (-2\vec{a}) = 0$.

Sol. (c) Two vectors are collinear if one is a scalar multiple of the other i.e., $\vec{b} = \lambda \vec{a}$.
Hence, $\vec{a}$ and $(-2\vec{a})$ are collinear (to be precise, they are parallel but they point in the opposite directions).

Also $\vec{a} \cdot (-2\vec{a}) = -2|\vec{a}|^2$. But Reason mentions $\vec{a} \cdot (-2\vec{a}) = 0$.

$\therefore$ Assertion is true but Reason is false.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. If $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$, find $\frac{dy}{dx}$ at $t = 2$.

Sol. We have $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$

$\Rightarrow \frac{dx}{dt} = 1 - \frac{1}{t^2} = \frac{t^2 - 1}{t^2}$ and $\frac{dy}{dt} = 1 + \frac{1}{t^2} = \frac{t^2 + 1}{t^2}$

As $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{t^2 + 1}{t^2} \times \frac{t^2}{t^2 - 1} = \frac{t^2 + 1}{t^2 - 1}$

At $t = 2$, $\frac{dy}{dx} = \frac{2^2 + 1}{2^2 - 1} = \frac{4 + 1}{4 - 1} = \frac{5}{3}$.

22. Find the sub-interval (s) of $\left(0, \frac{\pi}{2}\right)$ in which $f(x) = \tan x - 4x$ is increasing.

Sol. $f(x) = \tan x - 4x$

$\Rightarrow f'(x) = \sec^2 x - 4$

For $f'(x) = 0$, $\sec^2 x - 4 = 0 \Rightarrow \sec^2 x = 4 \Rightarrow \sec x = \pm 2$

$\because x \in \left(0, \frac{\pi}{2}\right) \therefore x = \frac{\pi}{3}$.

Note that, for $x \in \left(0, \frac{\pi}{3}\right]$, $f'(x) \le 0$. So, f is decreasing in this interval.

For $x \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$, $f'(x) \ge 0$. So, f is increasing in this interval.

Hence, the required sub-interval is $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$.

23. (a) Find the value of $\sin\left[\cot^{-1}\sqrt{2}\left(\cos(\tan^{-1}1)\right)\right]$.

OR

(b) A relation R on $A = \{1, 2, 3\}$ is defined as $R = \{(1,1), (3,3), (1,2)\}$. Is R a symmetric relation? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set $\{1, 2, 3\}$.

Sol. (a) $\sin\left[\cot^{-1}\sqrt{2}\left(\cos(\tan^{-1}1)\right)\right] = \sin\left[\cot^{-1}\sqrt{2}\left(\cos\frac{\pi}{4}\right)\right] = \sin\left[\cot^{-1}\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\right]$

$= \sin\left[\cot^{-1}1\right]$

$= \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

OR

(b) Since $(1,2) \in R$ but $(2,1) \notin R$. So, R is not symmetric.

If $R_1 = \{(2,2), (2,1)\}$, then $R \cup R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$.

Clearly, $R \cup R_1$ is reflexive, symmetric and transitive. Hence, it is an equivalence relation on the set $\{1, 2, 3\}$.

24. If for two unit vectors $\vec{a}$ and $\vec{b}$, $\left|\vec{a} + 2\vec{b}\right| = \left|2\vec{a} - \vec{b}\right|$, then find the angle between $\vec{a}$ and $\vec{b}$.

Sol. Let θ be the required angle between $\vec{a}$ and $\vec{b}$.

Now $|\vec{a}+2\vec{b}|=|2\vec{a}-\vec{b}| \Rightarrow |\vec{a}+2\vec{b}|^2=|2\vec{a}-\vec{b}|^2$

$\Rightarrow (\vec{a}+2\vec{b}).(\vec{a}+2\vec{b})=(2\vec{a}-\vec{b}).(2\vec{a}-\vec{b})$

$\Rightarrow |\vec{a}|^2+|2\vec{b}|^2+4\vec{a}.\vec{b}=|2\vec{a}|^2+|\vec{b}|^2-4\vec{a}.\vec{b}$

$\Rightarrow |\vec{a}|^2+4|\vec{b}|^2+4|\vec{a}||\vec{b}|\cos\theta=4|\vec{a}|^2+|\vec{b}|^2-4|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 1^2+4\times1^2+4\times1\times1\cos\theta=4\times1^2+1^2-4\times1\times1\cos\theta$

$\Rightarrow 5+4\cos\theta=5-4\cos\theta$

$\Rightarrow 8\cos\theta=0$

$\Rightarrow \cos\theta=0$

$\therefore \theta=\frac{\pi}{2}$.

25. (a) If the lines $\frac{x-3}{1}=\frac{1-y}{1}=\frac{z+2}{p}$ and $\frac{2-x}{3}=\frac{y+1}{5}=\frac{z+56}{2p}$ are perpendicular to each other, then find the value (s) of p.

OR

(b) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and $\vec{r}=\mu(\hat{i}-\hat{j}+\hat{k})$.

Sol. (a) Rewriting the given lines, we get $\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z+2}{p}$ and $\frac{x-2}{-3}=\frac{y+1}{5}=\frac{z+56}{2p}$.

Their d.r.'s are 1, –1, p and –3, 5, 2p respectively.

As the lines are perpendicular so, we shall use $a_1a_2+b_1b_2+c_1c_2=0$.

Therefore, $(1)(-3)+(-1)(5)+(p)(2p)=0$

$\Rightarrow -3-5+2p^2=0$

$\Rightarrow p^2=\frac{8}{2}$

$\Rightarrow p^2=4$

$\therefore p=\pm2$.

OR

(b) For the lines $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and $\vec{r}=\mu(\hat{i}-\hat{j}+\hat{k})$, the direction vectors are $3\hat{i}+4\hat{j}+2\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ respectively.

Since the required line is perpendicular to both the given lines, its direction is given by the cross product $(3\hat{i}+4\hat{j}+2\hat{k})\times(\hat{i}-\hat{j}+\hat{k})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&2\\1&-1&1\end{vmatrix}=6\hat{i}-\hat{j}-7\hat{k}$.

$\therefore$ Required vector equation of line is $\vec{r}=0\hat{i}+0\hat{j}+0\hat{k}+\lambda(6\hat{i}-\hat{j}-7\hat{k})$ i.e., $\vec{r}=\lambda(6\hat{i}-\hat{j}-7\hat{k})$.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. (a) Find $\int\frac{dx}{x^{1/2}+x^{1/3}}$.

OR

(b) Find $\int \tan^{-1}\left(\frac{1-x}{1+x}\right)dx$.

Sol. (a) Let $I=\int \frac{1}{x^{1/2}+x^{1/3}}dx$ [Put $x=t^6 \Rightarrow dx=6t^5dt$

$\therefore I=6\int \frac{t^5}{t^3+t^2}dt$

$\Rightarrow I=6\int \frac{t^3}{t+1}dt$

$\Rightarrow I=6\int\left(t^2-t+1-\frac{1}{t+1}\right)dt$

$\Rightarrow I=6\left(\frac{t^3}{3}-\frac{t^2}{2}+t-\log|t+1|\right)+C$

$\therefore I=2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log|\sqrt[6]{x}+1|+C$ or $I=2\sqrt{x}-3x^{1/3}+6x^{1/6}-6\log\left|x^{1/6}+1\right|+C$.

OR

(b) Let $I=\int \tan^{-1}\left(\frac{1-x}{1+x}\right)dx$ [Put $x=\tan\theta \Rightarrow dx=\sec^2 d\theta$

$\Rightarrow I=\int \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)\sec^2\theta d\theta$

$\Rightarrow I=\int \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)\sec^2\theta d\theta$

$\Rightarrow I=\int \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)\sec^2\theta d\theta$

$\Rightarrow I=\int\left(\frac{\pi}{4}-\theta\right)\sec^2\theta d\theta$

$\Rightarrow I=\left(\frac{\pi}{4}-\theta\right)\int \sec^2\theta d\theta-\int\left[\frac{d}{d\theta}\left(\frac{\pi}{4}-\theta\right)\int \sec^2\theta d\theta\right]d\theta$

$\Rightarrow I=\left(\frac{\pi}{4}-\theta\right)\tan\theta-\int -\tan\theta\, d\theta$

$\Rightarrow I=\left(\frac{\pi}{4}-\theta\right)\tan\theta+\log\left|\sec\theta\right|+C$

$\Rightarrow I=\frac{\pi}{4}x-x\tan^{-1}x+\log\left|\sqrt{1+x^2}\right|+C$ $\left[\because \sec\theta=\sqrt{1+\tan^2\theta}=\sqrt{1+x^2}\right.$

$\therefore I=\frac{\pi}{4}x-x\tan^{-1}x+\frac{1}{2}\log\left|1+x^2\right|+C$.

27. Evaluate $\int_0^{\pi}\frac{\sin^{2026}x}{\sin^{2026}x+\cos^{2026}x}dx$.

Sol. Let $I=\int_0^{\pi}\frac{\sin^{2026}x}{\sin^{2026}x+\cos^{2026}x}dx$

Consider $f(x)=\dfrac{\sin^{2026}x}{\sin^{2026}x+\cos^{2026}x}$

$\Rightarrow f(\pi-x)=\dfrac{\sin^{2026}(\pi-x)}{\sin^{2026}(\pi-x)+\cos^{2026}(\pi-x)}=\dfrac{\sin^{2026}x}{\sin^{2026}x+\cos^{2026}x}=f(x)$

$\therefore I=2\int_0^{\frac{\pi}{2}}\dfrac{\sin^{2026}x}{\sin^{2026}x+\cos^{2026}x}dx$ ...(i)

$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\dfrac{\sin^{2026}\left(\frac{\pi}{2}-x\right)}{\sin^{2026}\left(\frac{\pi}{2}-x\right)+\cos^{2026}\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I=2\int_0^{\frac{\pi}{2}}\dfrac{\cos^{2026}x}{\cos^{2026}x+\sin^{2026}x}dx$ ...(ii)

Adding (i) and (ii), we get $2I=2\int_0^{\frac{\pi}{2}}\left(\dfrac{\cos^{2026}x}{\cos^{2026}x+\sin^{2026}x}+\dfrac{\sin^{2026}x}{\cos^{2026}x+\sin^{2026}x}\right)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}1\,dx=[x]_0^{\pi/2}=\dfrac{\pi}{2}-0$

$\therefore I=\dfrac{\pi}{2}.$

28. (a) Find $\int\dfrac{\cos x}{(2+\sin x)(4+\sin x)}dx.$

OR

(b) Find $\int\dfrac{x+3}{x^2+4x+5}dx.$

Sol. (b) Put $(2+\sin x)=t\Rightarrow\cos x\,dx=dt$

$\therefore\int\dfrac{\cos x}{(2+\sin x)(4+\sin x)}dx=\int\dfrac{dt}{t(2+t)}=\int\dfrac{1}{2}\left[\dfrac{1}{t}-\dfrac{1}{(2+t)}\right]dt$

$=\dfrac{1}{2}\left[\log|t|-\log|2+t|\right]+C$

$=\dfrac{1}{2}\left[\log|2+\sin x|-\log|4+\sin x|\right]+C$

$=\dfrac{1}{2}\log\left|\dfrac{2+\sin x}{4+\sin x}\right|+C.$

OR

(b) Let $I=\int\dfrac{x+3}{x^2+4x+5}dx$

$\Rightarrow I=\dfrac{1}{2}\int\dfrac{(2x+4)+2}{x^2+4x+5}dx$

$\Rightarrow I=\dfrac{1}{2}\int\dfrac{(2x+4)}{x^2+4x+5}dx+\dfrac{1}{2}\int\dfrac{2}{x^2+4x+5}dx$ $\left[\because\int\dfrac{f'(x)}{f(x)}dx=\log|f(x)|+C\right.$

$$\Rightarrow I=\frac{1}{2}\log\left|x^2+4x+5\right|+\int\frac{1}{(x+2)^2+1^2}dx$$

$$\therefore I=\frac{1}{2}\log\left|x^2+4x+5\right|+\tan^{-1}(x+2)+C.$$

29. (a) Find the general solution of the differential equation $2x^2\frac{dy}{dx}=y^2+2xy$.

OR

(b) Find a particular solution of the differential equation $(x+1)\frac{dy}{dx}=2e^{-y}-1$, given that $y=0$ when $x=0$.

Sol. (a) Rewriting the D.E., we get $\frac{dy}{dx}=\frac{1}{2}\left(\frac{y}{x}\right)^2+\frac{y}{x}$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

So, $v+x\frac{dv}{dx}=\frac{1}{2}v^2+v$

$$\Rightarrow x\frac{dv}{dx}=\frac{1}{2}v^2$$

$$\Rightarrow\int\frac{2}{v^2}dv=\int\frac{dx}{x}$$

$$\Rightarrow-\frac{2}{v}=\log|x|+C$$

$$\Rightarrow-\frac{2x}{y}=\log|x|+C \text{ or } y=-\frac{2x}{\log|x|+C}.$$

OR

(b) Rewriting the D.E., we get $\frac{dy}{2e^{-y}-1}=\frac{dx}{x+1}$

$$\Rightarrow\int\frac{e^y dy}{2-e^y}=\int\frac{dx}{x+1}$$

$$\Rightarrow-\int\frac{-e^y dy}{2-e^y}=\int\frac{dx}{x+1} \qquad \left[\because\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+C\right.$$

$$\Rightarrow-\log\left|2-e^y\right|=\log|x+1|+\log|C|$$

$$\Rightarrow\log\left|\frac{1}{2-e^y}\right|=\log|C(x+1)|$$

$$\Rightarrow\frac{1}{2-e^y}=C(x+1)$$

It is given that $y=0$ when $x=0$. So, $\frac{1}{2-e^0}=C(0+1)$

$$\Rightarrow\frac{1}{2-1}=C \quad\Rightarrow C=1$$

So, the required particular solution is $\frac{1}{2-e^y}=(x+1)$.

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However we can further simplify it. Since $\frac{1}{2-e^y} = x+1$ implies $\frac{1}{x+1} = 2-e^y$

$\Rightarrow e^y = 2-\frac{1}{x+1}$ $\Rightarrow \log_e e^y = \log_e\left(\frac{2x+1}{x+1}\right)$ $\Rightarrow y\log_e e = \log_e\left(\frac{2x+1}{x+1}\right)$

$\therefore y = \log_e\left(\frac{2x+1}{x+1}\right)$ [$\because \log_e e = 1$

30. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

Sol. $(\sin x)^y = y^{\cos x}$

Taking log on both the sides, we get $\log(\sin x)^y = \log y^{\cos x}$

$\Rightarrow y\log(\sin x) = \cos x \log y$

On differentiating w.r.t. x, we get $y\times\frac{\cos x}{\sin x} + \log(\sin x)\times\frac{dy}{dx} = \cos x\times\frac{1}{y}\times\frac{dy}{dx} + \log y(-\sin x)$

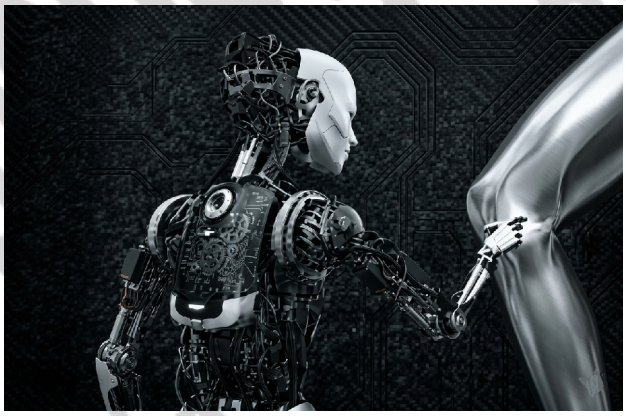
$\Rightarrow \log(\sin x)\times\frac{dy}{dx} - \frac{\cos x}{y}\times\frac{dy}{dx} = -\sin x\log y - y\times\cot x$

$\Rightarrow \left[\log(\sin x) - \frac{\cos x}{y}\right]\times\frac{dy}{dx} = -\left[\sin x\log y + y\times\cot x\right]$

$\Rightarrow \left[\frac{y\log(\sin x) - \cos x}{y}\right]\times\frac{dy}{dx} = -\left[\sin x\log y + y\times\cot x\right]$

$\therefore \frac{dy}{dx} = -\frac{y\left[\sin x\log y + y\times\cot x\right]}{y\log(\sin x) - \cos x}$ or $\frac{dy}{dx} = \frac{y\sin x\log y + y^2\cot x}{\cos x - y\log(\sin x)}$.

31. A survey was conducted on the patients who have undergone knee replacement surgeries.



It was found that, Robotic Knee replacement surgeries have 90% success rate.
On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that

(i) exactly one surgery is successful.
(ii) at most two surgeries are successful.

Sol. Let S represent a Success and F represent a Failure.

So, $P(S) = 90\% = 0.9$ and $P(F) = 0.1$.

(i) For exactly one surgery to be successful, there are three such scenarios i.e., SFF: Patient A succeeds, B fails, C fails; FSF: Patient A fails, B succeeds, C fails; FFS: Patient A fails, B fails, C succeeds.

$\therefore$ P(Exactly one success) = P(S)P(F)P(F) + P(F)P(S)P(F) + P(F)P(F)P(S)

$\Rightarrow$ P(Exactly one success) $= (0.9)(0.1)(0.1)\times 3 = 0.027 = \frac{27}{1000}$.

(ii) P(at most two successes) = 1 − P(all three succeeds) = 1 − P(S)P(S)P(S)

$= 1-(0.9)(0.9)(0.9) = 1-0.729 = 0.271 = \frac{271}{1000}$.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. Show that $f:\mathbb{R}\to\mathbb{R}$ defined as $f(x)=\frac{x}{\sqrt{1+x^2}}$ is one-one but not onto.

Sol. Let $f(x_1)=f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$ (domain).

Then $\frac{x_1}{\sqrt{1+x_1^2}} = \frac{x_2}{\sqrt{1+x_2^2}}$

$\Rightarrow x_1^2(1+x_2^2) = x_2^2(1+x_1^2)$

$\Rightarrow x_1^2(1+x_2^2) = x_2^2(1+x_1^2)$

$\Rightarrow x_1^2 + x_1^2x_2^2 = x_2^2 + x_1^2x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow |x_1| = |x_2|$

$\Rightarrow x_1 = x_2$ or $x_1 = -x_2$.

When $x_1 = -x_2$, then $f(x_1)=\frac{x_1}{\sqrt{1+x_1^2}}$ and $f(x_2)=f(-x_1)=\frac{-x_1}{\sqrt{1+x_1^2}}$.

If $f(x_1)=f(x_2)$, then $\frac{x_1}{\sqrt{1+x_1^2}} = \frac{-x_1}{\sqrt{1+x_1^2}}$ $\Rightarrow x_1 = -x_1$ $\therefore x_1 = 0$.

So, $f(x_1)=f(x_2)$ is possible only when $x_1 = 0$.

Thus $f(x_1)=f(x_2) \Rightarrow x_1 = x_2$.

Therefore, f is one-one.

Let $y = f(x) = \frac{x}{\sqrt{1+x^2}}$.

For f to be onto, for every $y \in \mathbb{R}$ (codomain), there must exist $x \in \mathbb{R}$ (domain).

Now $y^2 = \frac{x^2}{1+x^2}$

$\Rightarrow y^2 + x^2y^2 = x^2$

$\Rightarrow y^2 = x^2(1-y^2)$

$\Rightarrow x^2 = \frac{y^2}{1-y^2}$

For $x \in \mathbb{R}$, $x^2 = \frac{y^2}{1-y^2} \geq 0$ $\Rightarrow y^2 \geq 0$ [$\because 1-y^2 \neq 0$

Since $y^2 \geq 0$, so $\frac{y^2}{1-y^2} \geq 0$ is possible only when $1-y^2 > 0$ $\Rightarrow y^2 < 1$ $\Rightarrow -1 < y < 1$

This means the Range of f is (–1, 1).

Since the codomain is $\mathbb{R}$ and Range $\neq$ codomain.
So, f is not onto.

REMARK We can also get the range in following manner.

Note that, for any $x \in \mathbb{R}$, we have $x^2 < 1 + x^2$

$$\Rightarrow \frac{x^2}{1+x^2} < 1$$

$$\Rightarrow 0 \le \frac{x^2}{1+x^2} < 1$$

$$\Rightarrow 0 \le y^2 < 1$$

$$\Rightarrow -1 < y < 1.$$

33. Solve the following Linear Programming Problem graphically.

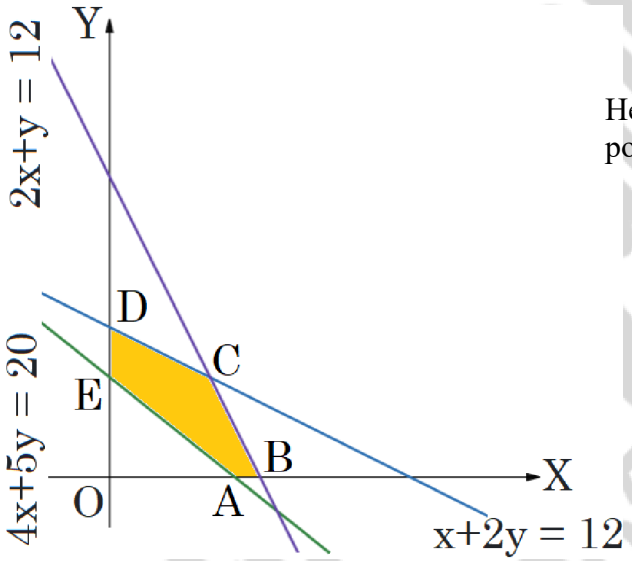
Maximise $Z = 600x + 400y$

Subject to the constraints $x + 2y \le 12$, $4x + 5y \ge 20$, $2x + y \le 12$; $x, y \ge 0$.

Sol. Consider the graph shown.

The corner points are A(5, 0), B(6, 0), C(4, 4), D(0, 6) and E(0, 4).

Therefore, $Z_A = 3000$, $Z_B = 3600$, $Z_C = 4000$, $Z_D = 2400$, $Z_E = 1600$.



Hence, the maximum value of Z is 4000 at the point (4, 4).

34. (a) On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹1000 and ₹500 were given to the lucky draw winners.



A total of 60 vouchers were given on the day. The number of ₹1000 vouchers added to 3 times the number of ₹500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

OR

(b) Given that $P=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $Q=\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$ and $R=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}$, find a matrix S such that $PQ-RS$ is a null matrix.

Sol. (a) Let x be the number of ₹1000 vouchers and y be the number of ₹500 vouchers.

According to the question, $x+y=60$, $x+3y=100$.

Let $A=\begin{bmatrix}1 & 1\\ 1 & 3\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}60\\ 100\end{bmatrix}$.

Note that, $AX=B$

$\Rightarrow A^{-1}AX=A^{-1}B$

$\Rightarrow IX=\frac{1}{3-1}\begin{bmatrix}3 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}60\\ 100\end{bmatrix}$ $\left[\because \text{For } A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\Rightarrow \text{adj.}A=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} \text{ and } A^{-1}=\frac{\text{adj.}A}{|A|}\right.$

$\Rightarrow X=\frac{1}{2}\begin{bmatrix}180-100\\ -60+100\end{bmatrix}=\frac{1}{2}\begin{bmatrix}80\\ 40\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}40\\ 20\end{bmatrix}$

By the def. of equality of matrices, we get $x=40$, $y=20$.

Hence, there are 40 vouchers of ₹1000 and 20 vouchers of ₹500.

OR

(b) Given $PQ-RS=O$

$\Rightarrow RS=PQ$

$\Rightarrow R^{-1}RS=R^{-1}PQ$ [Pre-multiplying with R^{-1} both the sides

$\Rightarrow IS=R^{-1}PQ$

$\Rightarrow S=\frac{1}{16-15}\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\ 7 & 4\end{bmatrix}$

$\left[\because \text{For } A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\Rightarrow \text{adj.}A=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} \text{ and } A^{-1}=\frac{\text{adj.}A}{|A|}\right.$

$\Rightarrow S=\frac{1}{1}\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}$

$\Rightarrow S=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$.

35. (a) Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.

$l_1: \frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$

$l_2: \frac{x+1}{-1}=\frac{2-y}{-2}=\frac{z-5}{5}$

OR

(b) Opposite sides of a square are along the lines :

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Find the area of the square. If direction ratios of other pairs of opposite sides of the square are given by $< -3, 6, p >$, find the value of p.

Sol. (a) For $l_1 : \dfrac{x+3}{-3} = \dfrac{y-1}{1} = \dfrac{z-5}{5}$ $\Rightarrow \vec{r} = -3\hat{i} + \hat{j} + 5\hat{k} + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$

For $l_2 : \dfrac{x+1}{-1} = \dfrac{2-y}{-2} = \dfrac{z-5}{5}$ $\Rightarrow \vec{r} = -\hat{i} + 2\hat{j} + 5\hat{k} + \lambda(-\hat{i} + 2\hat{j} + 5\hat{k})$

For the above lines, $\vec{a}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$, $\vec{b}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$ and $\vec{a}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}$, $\vec{b}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}$.

$$\Rightarrow (\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j},\ (\vec{b}_1 \times \vec{b}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -5\hat{i} + 10\hat{j} - 5\hat{k}.$$

Recall that two lines intersect if they are coplanar and not parallel.

Since $\dfrac{-3}{-1} \neq \dfrac{1}{2} \neq \dfrac{5}{5}$ i.e., d.r.'s for the lines are not proportional. It means, lines are not parallel.

Now we shall check if $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$ or not.

That is, $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (2\hat{i} + \hat{j}).(-5\hat{i} + 10\hat{j} - 5\hat{k}) = -10 + 10 + 0 = 0$.

Since $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$, the lines are coplanar. Furthermore, since the lines are not parallel. Therefore, the lines must intersect.

OR

(b) In a square, adjacent sides are perpendicular. The direction ratios of the first side are 2, 3, 6 and that of the other side are –3, 6, p.

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$, we get $(2)(-3) + (3)(6) + (6)(p) = 0$

$\Rightarrow -6 + 18 + (6p) = 0$

$\Rightarrow (6p) = -12$

$\therefore p = -2$.

Now the side length of the square is the perpendicular distance between the two parallel lines given (opposite sides are parallel in the square).

For the given lines, we have $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$.

So, $(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$ and $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$.

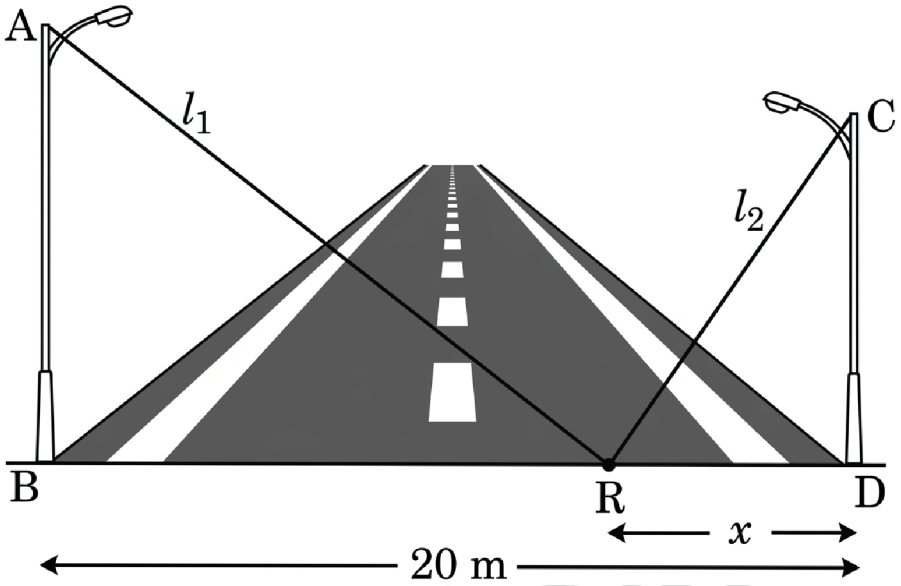
$$\therefore \text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} = \frac{\sqrt{81 + 196 + 16}}{\sqrt{4 + 9 + 36}} = \frac{\sqrt{293}}{\sqrt{49}} = \frac{\sqrt{293}}{7} \text{ units} = \text{Side length of square}.$$

Hence, area of square $= (\text{side})^2 = \left(\dfrac{\sqrt{293}}{7}\right)^2 = \dfrac{293}{49}$ Sq. units.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.



Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions.

(i) Express $p(x) = l_1 + l_2$ in terms of x.

(ii) Find $p'(x)$.

(iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum.

OR

(iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum?

Sol. **(i)** We shall use Pythagoras theorem in triangles CDR and ABR.

That is, $p(x) = l_1 + l_2 = \sqrt{22^2 + (20-x)^2} + \sqrt{16^2 + x^2} = \sqrt{484 + (20-x)^2} + \sqrt{256 + x^2}$.

(ii) $p'(x) = \dfrac{1}{2\sqrt{484 + (20-x)^2}} \times [-2(20-x)] + \dfrac{1}{2\sqrt{256 + x^2}} \times 2x$

$\Rightarrow p'(x) = \dfrac{x}{\sqrt{256 + x^2}} - \dfrac{(20-x)}{\sqrt{484 + (20-x)^2}}$.

(iii) (a) Let $f(x) = l_1^2 + l_2^2 = \left[484 + (20-x)^2\right] + \left[256 + x^2\right]$

$\Rightarrow f(x) = \left[484 + 400 - 40x + x^2\right] + \left[256 + x^2\right]$

$\Rightarrow f(x) = 2x^2 - 40x + 1140$

$\Rightarrow f'(x) = 4x - 40$ and $f''(x) = 4$

For $f'(x) = 0$, $4x - 40 = 0 \quad \Rightarrow x = 10$

Note that, $f''(10) = 4 > 0$. So, $f(x) = l_1^2 + l_2^2$ is minimum at $x = 10$.

OR

(iii) (b) Here the 22 m long pole is also replaced by a 16 m long pole.

Let $f(x) = \left[16^2 + (20-x)^2\right] + \left[16^2 + x^2\right]$

$\Rightarrow f(x) = \left[256 + 400 - 40x + x^2\right] + \left[256 + x^2\right]$

$\Rightarrow f(x) = 2x^2 - 40x + 912$

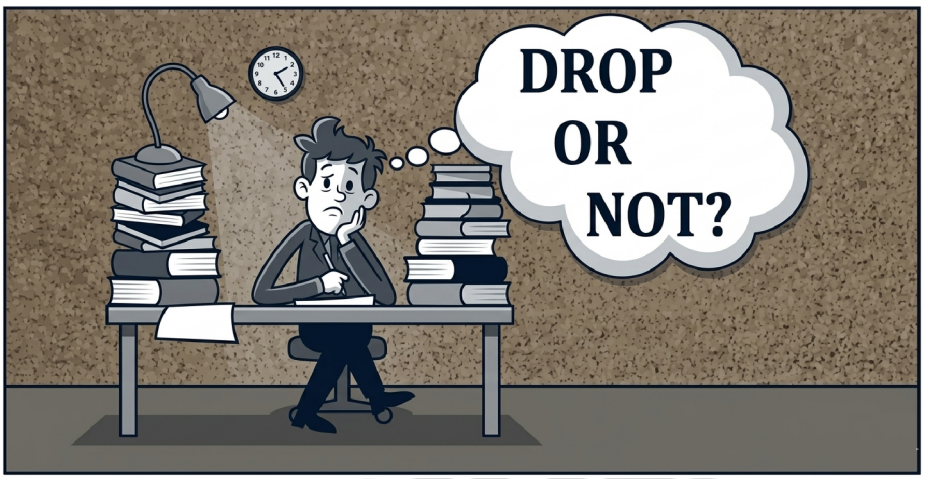
$\Rightarrow f'(x) = 4x - 40$ and $f''(x) = 4$

For $f'(x) = 0$, $4x - 40 = 0 \quad \Rightarrow x = 10$

Note that, $f''(10) = 4 > 0$. So, $f(x)$ is minimum at $x = 10$.

The ladders should be kept exactly in the middle of the road, 10 m from either pole, to minimize the sum of the squares of their lengths.

37. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.



As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.

Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.

Based on the given information, answer the following questions.

(i) Find the probability that a student selected at random is a regular student.

(ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination?

(iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout.

OR

(iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.

Sol. Let E_1 and E_2 be the events that a student is a dropout and that a student is a regular student, respectively. Let A be the event that a student qualifies the examination.

Now $P(E_1) = 40\%$, $P(E_2) = 1 - P(E_1) = 60\%$, $P(A \mid E_1) = 5\%$, $P(A \mid E_2) = 10\%$.

(i) $P(E_2) = 60\% = 0.60$.

(ii) $P(A' \mid E_1) = 1 - P(A \mid E_1) = 1 - 5\% = 95\% = 0.95$.

(iii) (a) Using Bayes' theorem, $P(E_2 \mid A) = \dfrac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$

$$\Rightarrow P(E_2 \mid A) = \frac{0.6 \times 0.1}{0.4 \times 0.05 + 0.6 \times 0.1}$$

$$\Rightarrow P(E_2 \mid A) = \frac{6}{2+6} = \frac{6}{8} = \frac{3}{4} = 75\% \text{ or } 0.75.$$

OR

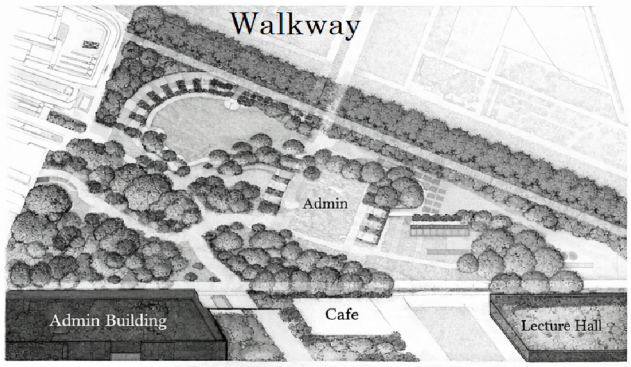
(iii) (b) Here $P(A' \mid E_1) = 1 - P(A \mid E_1) = 1 - 5\% = 95\% = 0.95$,

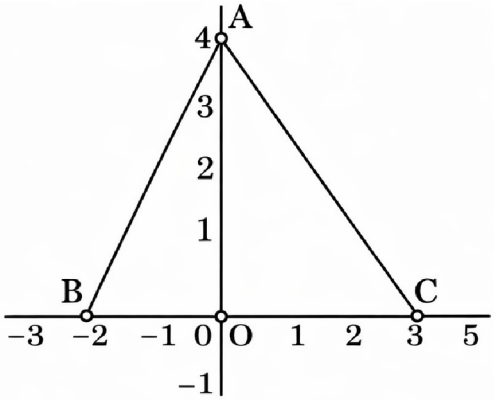
$P(A' \mid E_2) = 1 - P(A \mid E_2) = 1 - 10\% = 90\% = 0.90$.

Using Bayes' theorem, $P(E_2 \mid A') = \dfrac{P(E_2)P(A' \mid E_2)}{P(E_1)P(A' \mid E_1) + P(E_2)P(A' \mid E_2)}$

$\Rightarrow P(E_2 \mid A') = \dfrac{0.6 \times 0.90}{0.4 \times 0.95 + 0.6 \times 0.90} = \dfrac{54}{38+54} = \dfrac{54}{92} = \dfrac{27}{46}$.

38. There is a triangular park in the society. The park is divided into two sections as shown in the figure.





In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed. The vertices of the triangular park ABC are A(0, 4), B(–2, 0) and C(3, 0).

Based on the above information, answer the following questions.

(i) Write the equation of the boundary line AB of the park.

(ii) Write the equation of the boundary line AC of the park.

(iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football.

OR

(iii) (b) Using integration, find the area of region AOB.

Sol. We have A(0, 4), B(–2, 0) and C(3, 0).

(i) Equation of line AB : $\dfrac{y-4}{x-0} = \dfrac{0-4}{-2-0} \quad \Rightarrow \dfrac{y-4}{x} = 2$ or $2x - y + 4 = 0$.

(ii) Equation of line AC : $\dfrac{y-4}{x-0} = \dfrac{0-4}{3-0} \quad \Rightarrow \dfrac{y-4}{x} = \dfrac{-4}{3}$ or $4x + 3y - 12 = 0$.

(iii) (a) Required area of region $OAC = \displaystyle\int_0^3 \frac{12-4x}{3}\,dx = \frac{4}{3}\int_0^3 (3-x)\,dx$

$$= \frac{4}{3}\left[\frac{(3-x)^2}{2(-1)}\right]_0^3$$

$$= -\frac{2}{3}[0-9] = 6 \text{ Sq. units}.$$

OR

(iii) (b) Required area of region $AOB = \displaystyle\int_{-2}^{0} (2x+4)\,dx$

$$= \left[\frac{(2x+4)^2}{2(2)}\right]_{-2}^{0}$$

$$= \frac{1}{4}[16-0] = 4 \text{ Sq. units}.$$

☑ Q.P. Code - 65/3/2

01. If $\tan^{-1} x = y$, then $\frac{dy}{dx}$ is equal to

(a) $(\sec^{-1} x)^2$ (b) $\sec^2 y$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\cos^2 y$

Sol. (d) $\tan^{-1} x = y \Rightarrow \frac{1}{1+x^2} = \frac{dy}{dx}$

$\Rightarrow \frac{1}{1+\tan^2 y} = \frac{dy}{dx}$ [$\because \tan^{-1} x = y \Rightarrow x = \tan y$

$\Rightarrow \frac{1}{\sec^2 y} = \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \cos^2 y$.

03. $\int \frac{dx}{\sqrt{e^{-2x}-1}}$ is equal to

(a) $\sin^{-1} e^{-x} + C$ (b) $\log\left|e^{-x} + \sqrt{e^{-2x}-1}\right| + C$

(c) $\sin^{-1} e^{x} + C$ (d) $\log\left|e^{-x} - \sqrt{e^{-2x}-1}\right| + C$

Sol. (c) $\int \frac{dx}{\sqrt{e^{-2x}-1}} = \int \frac{dx}{\sqrt{\frac{1}{e^{2x}}-1}} = \int \frac{e^x\,dx}{\sqrt{1-e^{2x}}}$ [Put $e^x = t \Rightarrow e^x\,dx = dt$

$= \int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} t + C = \sin^{-1} e^x + C$.

10. Direction ratios of lines l_1 and l_2 respectively are $<1, -2, 3>$ and $<-2, p, -6>$. The values of p and q for which $l_1 \| l_2$ is

(a) –4 (b) 4 (c) –10 (d) 10

Sol. (b) For parallel lines, their d.r.'s are proportional.

$\therefore \frac{1}{-2} = \frac{-2}{p} = \frac{3}{-6}$

Consider $\frac{1}{-2} = \frac{-2}{p} \Rightarrow p = 4$.

12. The domain of $\sin^{-1}(1-2x)$ is

(a) $[-1, 1]$ (b) $[-1, 3]$ (c) $[-2, 2]$ (d) $[0, 1]$

Sol. (d) $\sin^{-1}(1-2x)$ is defined when $-1 \le (1-2x) \le 1$

$\Rightarrow -1-1 \le (1-2x)-1 \le 1-1$

$\Rightarrow -2 \le (-2x) \le 0$

$\Rightarrow 1 \ge x \ge 0$

$\therefore$ Domain $= x \in [0, 1]$.

15. If a matrix B is such that $B\begin{bmatrix}1 & 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 1\\ 2 & 0 & 1\end{bmatrix}$, then the order of matrix B is

(a) 1×3 (b) 3×1 (c) 3×3 (d) 1×1

Sol. (b) $\because B_{m\times n}\begin{bmatrix}1 & 2 & 3\end{bmatrix}_{1\times3}=\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 1\\ 2 & 0 & 1\end{bmatrix}_{3\times3}$ $\therefore n=1,\ m=3$

Hence, order of matrix B is 3×1.

17. If $A(\text{adj.}A)=\begin{bmatrix}2026 & 0 & 0\\ 0 & 2026 & 0\\ 0 & 0 & 2026\end{bmatrix}$, then the value of $|\text{adj.}A|$ is equal to

(a) 2026 (b) $(2026)^{-1}$ (c) $(2026)^{-2}$ (d) $(2026)^2$

Sol. (d) Since $A(\text{adj.}A)=|A|I=2026\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. So, $|A|=2026$.

Now $|\text{adj.}A|=|A|^{3-1}=(2026)^2$.

24. If $x=e^{t+\frac{1}{t}}$, $y=e^{t-\frac{1}{t}}$, then find $\frac{dy}{dx}$ at $t=-2$.

Sol. $x=e^{t+\frac{1}{t}}$, $y=e^{t-\frac{1}{t}}$

Differentiating w.r.t. t, we get $\frac{dx}{dt}=e^{t+\frac{1}{t}}\left(1-\frac{1}{t^2}\right)=e^{t+\frac{1}{t}}\left(\frac{t^2-1}{t^2}\right)$, $\frac{dy}{dt}=e^{t-\frac{1}{t}}\left(1+\frac{1}{t^2}\right)=e^{t-\frac{1}{t}}\left(\frac{t^2+1}{t^2}\right)$.

As $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$

$\Rightarrow\frac{dy}{dx}=e^{t-\frac{1}{t}}\left(\frac{t^2+1}{t^2}\right)\times\left(\frac{t^2}{t^2-1}\right)e^{-t-\frac{1}{t}}=\frac{t^2+1}{t^2-1}\times e^{\frac{-2}{t}}$

Now $\frac{dy}{dx}\left(\text{at } t=-2\right)=\frac{(-2)^2+1}{(-2)^2-1}\times e^{\frac{-2}{(-2)}}=\frac{5e}{3}$.

25. Find the sub-interval of $(0,\infty)$ in which $f(x)=x^2e^{-x}$ is increasing.

Sol. $f(x)=x^2e^{-x}\ \Rightarrow f'(x)=x^2e^{-x}(-1)+e^{-x}(2x)=xe^{-x}(2-x)$

For $f'(x)=0$, $xe^{-x}(2-x)=0$

Since $e^{-x}\neq0$ $\therefore x=0$ or $(2-x)=0$

$\Rightarrow x=2\in(0,\infty)$

For $x\in(0,2]$, $f'(x)\geq0$. So, f is increasing in $(0,2]$.

For $x\in[2,\infty)$, $f'(x)\leq0$. So, f is decreasing in $[2,\infty)$.

Hence, the required sub-interval of $(0,\infty)$ in which f is increasing is $(0,2]$.

27. (a) Find the general solution of the differential equation $(x^2+y^2)dy=xydx$.

OR

(b) Find the particular solution of the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$.

Sol. (a) Rewriting the D.E., we get $\frac{dx}{dy} = \frac{(x^2 + y^2)}{xy} \quad \Rightarrow \frac{dx}{dy} = \frac{x}{y} + \frac{y}{x}$

Put $x = vy \quad \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore v + y\frac{dv}{dy} = v + \frac{1}{v}$

$\Rightarrow y\frac{dv}{dy} = \frac{1}{v} \quad \Rightarrow \int v\,dv = \int \frac{dy}{y}$

$\Rightarrow \frac{1}{2}v^2 = \log|y| + C$

$\Rightarrow \frac{x^2}{2y^2} = \log|y| + C$.

OR

(b) We shall compare the given D.E. with $\frac{dy}{dx} + P(x)y = Q(x)$.

So, we get $P(x) = -3\cot x$, $Q(x) = \sin 2x$.

Now I.F. $= e^{\int -3\cot x\,dx} = e^{-3\log\sin x} = \frac{1}{e^{\log\sin^3 x}} = \frac{1}{\sin^3 x}$.

Then the general solution will be $\frac{1}{\sin^3 x} \times y = \int \frac{1}{\sin^3 x} \times \sin 2x\,dx + C$

$\Rightarrow \frac{y}{\sin^3 x} = \int \frac{1}{\sin^3 x} \times 2\sin x\cos x\,dx + C$

$\Rightarrow \frac{y}{\sin^3 x} = 2\int \operatorname{cosec} x\cot x\,dx + C$

$\Rightarrow \frac{y}{\sin^3 x} = -2\operatorname{cosec} x + C$

$\Rightarrow y = -2\sin^2 x + C\sin^3 x$

It is given that $y = 2$ when $x = \frac{\pi}{2}$. So, $2 = -2\sin^2\frac{\pi}{2} + C\sin^3\frac{\pi}{2} \quad \Rightarrow C = 4$.

Therefore, the required particular solution is $y = -2\sin^2 x + 4\sin^3 x$.

31. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{2^x + 1}dx$.

Sol. Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{2^x + 1}dx$ ...(i)

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right)}{2^{\left(-\frac{\pi}{2}+\frac{\pi}{2}-x\right)}+1}dx$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos^2(-x)}{2^{-x}+1}dx$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{2^x\cos^2 x}{1+2^x}dx \quad ...(ii)$$

Adding (i) and (ii), we get $2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+2^x}{1+2^x}\cos^2 x\,dx$

$$\Rightarrow 2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 x\,dx$$

$\left[\text{Taking } f(x)=\cos^2 x\Rightarrow f(-x)=\cos^2(-x)=\cos^2 x=f(x)\right.$
$\left.\therefore f \text{ is even function}\right]$

$$\Rightarrow 2I=2\int_0^{\frac{\pi}{2}}\cos^2 x\,dx=\int_0^{\frac{\pi}{2}}(1+\cos 2x)\,dx$$

$$\Rightarrow I=\frac{1}{2}\left[x+\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow I=\frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-\left(0+\frac{1}{2}\sin 0\right)\right]$$

$$\therefore I=\frac{\pi}{4}.$$

34. Show that $f:\mathbb{R}_+\to[-5,\infty)$ given by $f(x)=4x^2+4x-5$ is both one-one and onto where $\mathbb{R}_+=[0,\infty)$. Also, find $p\in\mathbb{R}_+$, such that $f(p)=3$.

Sol. Let $f(x_1)=f(x_2)$ for some $x_1,x_2\in\mathbb{R}_+$ (domain).

Then $4x_1^2+4x_1-5=4x_2^2+4x_2-5$

$\Rightarrow 4x_1^2-4x_2^2+4x_1-4x_2=0$

$\Rightarrow 4(x_1-x_2)(x_1+x_2)+4(x_1-x_2)=0$

$\Rightarrow 4(x_1-x_2)(x_1+x_2+1)=0$

$\Rightarrow (x_1-x_2)=0$ [$\because x_1, x_2\geq 0\ \therefore (x_1+x_2+1)\geq 1$ and it can't be 0

$\Rightarrow x_1=x_2$

$\therefore f$ is one-one.

Let $y\in[-5,\infty)$ such that $y=f(x)$.

That is, $y=4x^2+4x-5$

$\Rightarrow y=(2x+1)^2-6$

$\Rightarrow y+6=(2x+1)^2$

$\Rightarrow 2x+1=\sqrt{y+6}$ [Taking positive square root as $x\geq 0$

$\Rightarrow x = \dfrac{\sqrt{y+6}-1}{2}$

For any $y \geq -5$, $y+6 \geq 1$, so $\sqrt{y+6} \geq 1$. This implies, $x \geq 0$ which belongs to the domain $\mathbb{R}_+$.

Since every element in the codomain has a pre-image in the domain, f is onto.

Now taking $f(p) = 3$, we get $4p^2 + 4p - 5 = 3$

$\Rightarrow 4p^2 + 4p - 8 = 0 \quad \Rightarrow p^2 + p - 2 = 0$

$\Rightarrow (p-1)(p+2) = 0$

Either $(p-1) = 0$ or $(p+2) = 0$

$\therefore p = 1, -2$.

But $p \in \mathbb{R}_+$. Hence, $p = 1$.

Alternatively, we can use derivative approach also.

We have $f(x) = 4x^2 + 4x - 5 \Rightarrow f'(x) = 8x + 4 > 0 \ \forall \ x \in \mathbb{R}_+$.

Therefore f (x) is **strictly increasing** on [0, ∞). Hence f is one-one.

Since the minimum value occurs at the smallest value of the domain i.e., at $x = 0$.

So, minimum value of f is $f(0) = 4 \times 0^2 + 4 \times 0 - 5 = -5$.

As the function increases for all $x \in \mathbb{R}_+ = [0, \infty)$, therefore the $\text{Range}(f) = [-5, \infty) = \text{Codomain}$.

Hence, f is onto.

35. Solve the following Linear Programming Problem graphically.

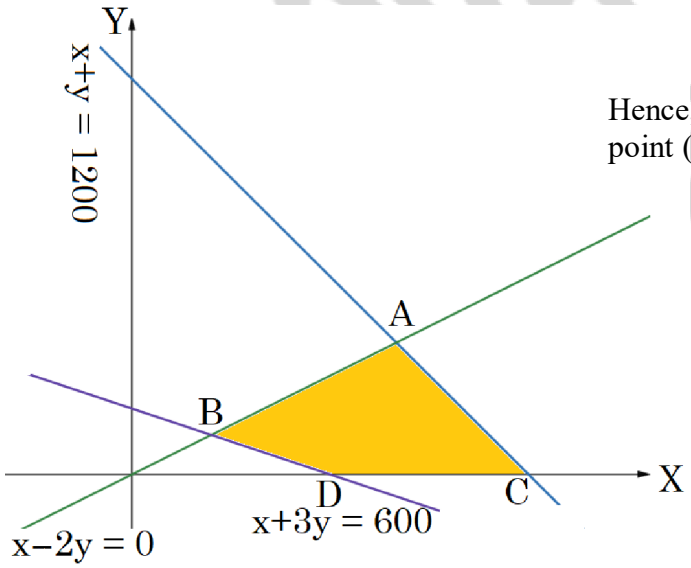
Maximise $Z = 12x + 18y$

Subject to the constraints $x + y \leq 1200$, $x - 2y \geq 0$, $x + 3y \geq 600$, $x \geq 0$, $y \geq 0$.

Sol. Consider the graph shown.

The corner points are A(800, 400), B(240, 120), C(1200, 0) and D(600, 0).

Therefore, $Z_A = 16800$, $Z_B = 5040$, $Z_C = 14400$, $Z_D = 7200$.



Hence, the maximum value of Z is 16800 at the point (800, 400).

☑ Q.P. Code - 65/3/3

03. Direction ratios of lines l_1 and l_2 respectively are $<1, 0, 0>$ and $<0, -1, 0>$. The direction ratios of the line perpendicular to both l_1 and l_2 is

(a) $<1, 1, 0>$ (b) $<0, 0, -1>$ (c) $<1, 1, 1>$ (d) $<1, 0, -1>$

Sol. (b) The direction vectors along the lines l_1 and l_2 respectively are $\vec{b}_1 = \hat{i}$ and $\vec{b}_2 = -\hat{j}$.

Therefore, the direction vector of required line will be $\vec{b}_1 \times \vec{b}_2 = (\hat{i}) \times (-\hat{j}) = -\hat{k}$.

Hence, the required d.r.'s are 0, 0, –1.

05. The domain of $\cos^{-1}(4x+1)$ is

(a) $[-1, 1]$ (b) $[-3, 5]$ (c) $[-4, 4]$ (d) $\left[-\frac{1}{2}, 0\right]$

Sol. (d) $\cos^{-1}(4x+1)$ is defined when $-1 \le (4x+1) \le 1$

$\Rightarrow -1-1 \le (4x+1)-1 \le 1-1$

$\Rightarrow -2 \le (4x) \le 0$

$\Rightarrow -\frac{1}{2} \le x \le 0$

$\therefore$ Domain $= x \in \left[-\frac{1}{2}, 0\right]$.

08. If a matrix X is such that $\begin{bmatrix}2 & 1\end{bmatrix} X = \begin{bmatrix}3 & 4 & 5\end{bmatrix}$, then the order of matrix X is

(a) 3×1 (b) 2×3 (c) 1×2 (d) 1×3

Sol. (b) Since $\begin{bmatrix}2 & 1\end{bmatrix}_{1\times2} X_{m\times n} = \begin{bmatrix}3 & 4 & 5\end{bmatrix}_{1\times3}$. So, $2 = m$, $n = 3$.

Therefore, the order of matrix X is 2×3.

10. If $A(\text{adj.}A) = \begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$, then the value of $|2A|$ is

(a) 6 (b) 54 (c) 12 (d) 24

Sol. (d) Since $A(\text{adj.}A) = |A|I = 3\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. So, $|A| = 3$.

$\therefore\ |2A| = 2^3|A| = 8\times3 = 24$.

12. If $2\cos^{-1}x = y$, then $\frac{dy}{dx}$ is

(a) $-2\sin^{-1}x$ (b) $-\frac{1}{2}\sin\frac{y}{2}$ (c) $\frac{-1}{\sqrt{1-2x^2}}$ (d) $-2\,\text{cosec}\,\frac{y}{2}$

Sol. (d) $2\cos^{-1}x = y \quad \Rightarrow -\frac{2}{\sqrt{1-x^2}} = \frac{dy}{dx}$

$\Rightarrow -\frac{2}{\sqrt{1-\cos^2\frac{y}{2}}} = \frac{dy}{dx}$ $\left[\because 2\cos^{-1}x = y \Rightarrow x = \cos\frac{y}{2}\right.$

$\Rightarrow -\frac{2}{\sqrt{\sin^2\frac{y}{2}}} = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = -\frac{2}{\sin\frac{y}{2}}$

$\therefore \frac{dy}{dx} = -2\,\text{cosec}\,\frac{y}{2}$.

14. $\int \frac{\cos x}{\sqrt{\sin^2 x+1}}dx$ is equal to

(a) $\tan^{-1}(\sin x)+C$ (b) $\sin^{-1}(\sin x)+C$

(c) $\log\left|\sin x+\sqrt{1+\sin^2 x}\right|+C$ (d) $\log\left|\sin x-\sqrt{\sin^2 x+1}\right|+C$

Sol. (c) Put $\sin x = t \Rightarrow \cos x\,dx = dt$.

$$\therefore \int \frac{\cos x}{\sqrt{\sin^2 x+1}}dx = \int \frac{1}{\sqrt{t^2+1}}dt = \log\left|t+\sqrt{t^2+1}\right|+C = \log\left|\sin x+\sqrt{1+\sin^2 x}\right|+C.$$

22. If $x = e^{\sin^{-1}t}$, $y = e^{\cos^{-1}t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.

Sol. $x = e^{\sin^{-1}t}$, $y = e^{\cos^{-1}t}$

Differentiating w.r.t. t, we get $\frac{dx}{dt} = e^{\sin^{-1}t}\times\frac{1}{\sqrt{1-t^2}}$, $\frac{dy}{dt} = e^{\cos^{-1}t}\times\left(-\frac{1}{\sqrt{1-t^2}}\right)$

As $\frac{dy}{dx} = \frac{dy}{dt}\times\frac{dt}{dx}$ so, $\frac{dy}{dx} = e^{\cos^{-1}t}\times\left(-\frac{1}{\sqrt{1-t^2}}\right)\times\frac{\sqrt{1-t^2}}{e^{\sin^{-1}t}}$

$$\Rightarrow \frac{dy}{dx} = -e^{\cos^{-1}t-\sin^{-1}t}$$

At $t = \frac{1}{\sqrt{2}}$, $\frac{dy}{dx} = -e^{\cos^{-1}\frac{1}{\sqrt{2}}-\sin^{-1}\frac{1}{\sqrt{2}}} = -e^{\frac{\pi}{4}-\frac{\pi}{4}} = -e^0 = -1$.

23. Find the sub-interval of $(0,\infty)$ in which $f(x) = x\log x$ is increasing.

Sol. We have $f(x) = x\log x \Rightarrow f'(x) = x\times\frac{1}{x}+\log x\times 1 = 1+\log x$

Now the function f is increasing when $f'(x) > 0$, $1+\log x > 0$

$$\Rightarrow \log x > -1$$

$$\Rightarrow x > e^{-1} \text{ or } x > \frac{1}{e}$$

Hence, the sub-interval of $(0,\infty)$ in which $f(x)$ is increasing is $\left(\frac{1}{e},\infty\right)$.

28. Find $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x}dx$.

Sol. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x}dx$ ...(i)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{1+\sin 2\left(\frac{\pi}{2}-x\right)}dx = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+\sin(\pi-2x)}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+\sin 2x}dx \quad ...(ii)$$

Adding (i) and (ii), we get $2I=\int_0^{\frac{\pi}{2}}\left[\frac{\sin^2 x}{1+\sin 2x}+\frac{\cos^2 x}{1+\sin 2x}\right]dx=\int_0^{\frac{\pi}{2}}\left[\frac{1}{1+\sin 2x}\right]dx$

$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{1}{1+\sin 2x}dx$

$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{1}{(\cos x+\sin x)^2}dx$

$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{1}{2\left[\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right]^2}dx$

$\Rightarrow I=\frac{1}{2}\int_0^{\frac{\pi}{2}}\frac{1}{\left[\cos\frac{\pi}{4}\cos x+\sin\frac{\pi}{4}\sin x\right]^2}dx=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{1}{\cos^2\left(x-\frac{\pi}{4}\right)}dx$

$\Rightarrow I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\sec^2\left(x-\frac{\pi}{4}\right)dx=\frac{1}{4}\left[\tan\left(x-\frac{\pi}{4}\right)\right]_0^{\frac{\pi}{2}}$

$\Rightarrow I=\frac{1}{4}\left[\tan\left(\frac{\pi}{2}-\frac{\pi}{4}\right)-\tan\left(0-\frac{\pi}{4}\right)\right]=\frac{1}{4}[1-(-1)]$

$\therefore I=\frac{1}{2}.$

30. (a) Find the general solution of the differential equation $(x^2-y^2)dx+2xydy=0$.

OR

(b) Solve the differential equation

$\sin x\cos y\,dx+\cos x\sin y\,dy=0$, given that $y=\frac{\pi}{4}$ when $x=0$.

Sol. (a) Rewriting the D.E., we get $2\frac{dy}{dx}=\frac{y^2-x^2}{xy}$ $\Rightarrow 2\frac{dy}{dx}=\frac{y}{x}-\frac{x}{y}$

Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore 2\left(v+x\frac{dv}{dx}\right)=v-\frac{1}{v}$

$\Rightarrow 2v+2x\frac{dv}{dx}=v-\frac{1}{v}$

$\Rightarrow 2x\frac{dv}{dx}=-v-\frac{1}{v}$

$\Rightarrow 2x\frac{dv}{dx}=-\frac{v^2+1}{v}$

$\Rightarrow\int\frac{2v}{v^2+1}dv=-\int\frac{dx}{x}$

$\Rightarrow\log|v^2+1|=-\log|x|+\log|C|$

$\Rightarrow \log\left|\frac{y^2}{x^2}+1\right| = \log\left|\frac{C}{x}\right|$

$\therefore \frac{y^2}{x^2}+1 = \frac{C}{x}$ or $y^2+x^2 = xC$ is the required general solution.

OR

(b) Rewriting the D.E., we get $\frac{\sin x}{\cos x}dx + \frac{\sin y}{\cos y}dy = 0$

$\Rightarrow \int \tan x\,dx + \int \tan y\,dy = 0$

$\Rightarrow \log|\sec x| + \log|\sec y| = \log|C|$

$\Rightarrow \log|\sec x \sec y| = \log|C|$

$\Rightarrow \sec x \sec y = C$

It is given that $y = \frac{\pi}{4}$ when $x = 0$. So, $\sec 0 \sec\frac{\pi}{4} = C \quad \Rightarrow C = \sqrt{2}$

Hence, the required particular solution is $\sec x \sec y = \sqrt{2}$ or $\sqrt{2}\cos x \cos y = 1$.

33. Show that a function $f : \mathbb{R}_+ \to A \subset \mathbb{R}$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $\mathbb{R}_+ = [0, \infty)$. Also, find if there exists $a \in \mathbb{R}_+$, such that $f(a) = 7$. Justify.

Sol. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}_+$ (domain).

Then $4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow 4x_1^2 - 4x_2^2 + 12x_1 - 12x_2 = 0$

$\Rightarrow 4(x_1 - x_2)(x_1 + x_2) + 12(x_1 - x_2) = 0$

$\Rightarrow 4(x_1 - x_2)(x_1 + x_2 + 3) = 0$

$\Rightarrow (x_1 - x_2) = 0 \qquad [\because x_1, x_2 \in [0, \infty) \therefore 4(x_1 + x_2 + 3) \geq 3$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

Now for f to be onto, set A must be equal to the range of f.

So, $f(x) = 4x^2 + 12x + 15 = (2x+3)^2 + 6$.

Since $x \in [0, \infty) \quad \Rightarrow 0 \leq x < \infty \quad \Rightarrow 0 \leq 2x < \infty$

$\Rightarrow 3 \leq 2x + 3 < \infty \quad \Rightarrow 9 \leq (2x+3)^2 < \infty$

$\Rightarrow 9 + 6 \leq (2x+3)^2 + 6 < \infty$

$\Rightarrow 15 \leq f(x) < \infty$

It seems that the range of $f(x)$ is $[15, \infty)$.

$\therefore A = [15, \infty)$.

But $A \subset \mathbb{R}$. That is, A is a subset of real numbers ($\mathbb{R}$).

Hence, Range $(f) = A = [15, \infty)$.

Clearly, for every value from $A = [15, \infty)$, there exists some value in the domain $x \in [0, \infty)$, making f an onto function.

For $f(a) = 7$, as the minimum value of $f(x)$ for $x \in [0, \infty)$ is 15, and $7 < 15$, there is **no value** of 'a' in the domain that maps to 7.

Justification : $f(a) = 7$, $4a^2 + 12a + 15 = 7 \quad \Rightarrow 4a^2 + 12a + 8 = 0$

$\Rightarrow a^2 + 3a + 2 = 0$

$\Rightarrow (a+1)(a+2)=0$

$\Rightarrow a=-1, -2 \notin R_+$.

REMARK In the question paper, the original function was defined as $f:\mathbb{R}_+ \to A \subset N$. In this codomain, f can not be a function. Because for every $x \in \mathbb{R}_+$, we do not have image in the given codomain $A \subset N$. For example, $f(0.1)=16.24 \notin A \subset N$. We've done necessary changes while solving this question.

34. Solve the following Linear Programming Problem graphically.

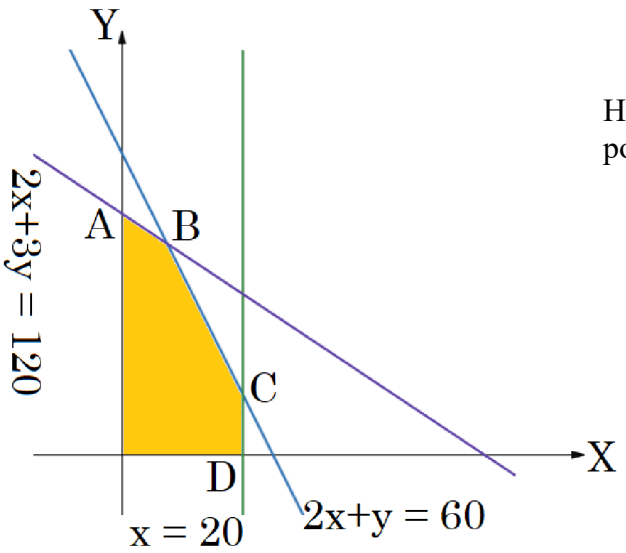
Maximise $Z=8x+8y$

Subject to the constraints $x-20 \le 0$, $2x+3y \le 120$, $2x+y \le 60$, $x \ge 0$, $y \ge 0$.

Sol. Consider the graph shown.

The corner points are O(0, 0), A(0, 40), B(15, 30), C(20, 20) and D(20, 0).

Therefore, $Z_O=0$, $Z_A=320$, $Z_B=360$, $Z_C=320$, $Z_D=160$.



Hence, the maximum value of Z is 360 at the point (15, 30).

With regards,
O.P. GUPTA
Math Mentor
Author - MATHMISSION Series
Indira Award Winner
The O.P. Gupta Advanced Math Classes
@ **Think Academy**, Najafgarh
Telegram / WhatsApp : +919650350480
YouTube.com/@theopgupta

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Kindly forward your concerns/feedbacks through **message** on **WhatsApp @ +919650350480** or mail at **iMathematicia@gmail.com**

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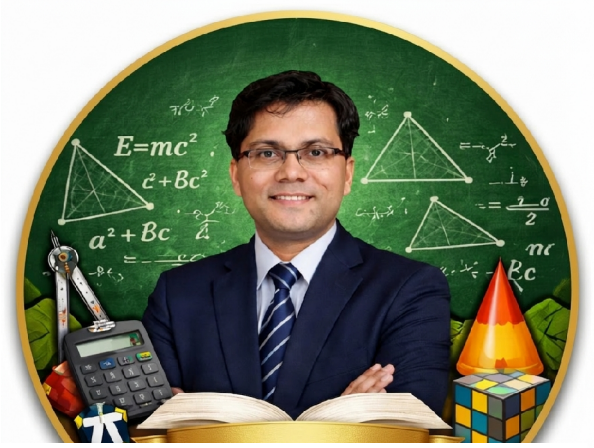
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O.P. Gupta
Author - Mathmission Series of Books
Founder & Mentor
THE O.P. GUPTA ADVANCED MATH CLASSES
@ Thana Road, Najafgarh, New Delhi

WhatsApp: +91 9650350480



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